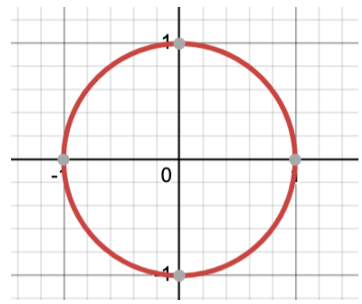
5.3i Graphing the Sine and Cosine Function part i

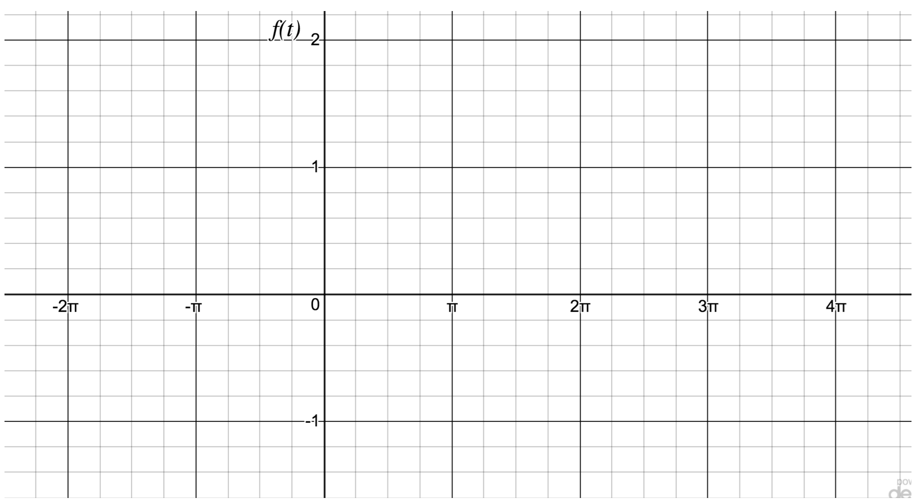
$$f(t) = \sin(t)$$

t_1	$f(t_1)$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	

t_1	$f(t_1)$
π	
$\frac{3\pi}{2}$	
2π	



Note choice of scale on t axis.



Picture the unit circle. What happens to the ________________ values as t increases from $0 \to \pi/2$

What does y do as t goes from

$\pi/2 \to \pi$ _________________________

$\pi \to 3\pi/2$ _________________________

$3\pi/2 \to 2\pi$ _________________________

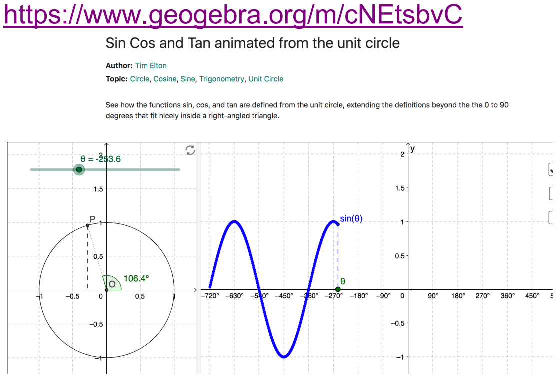
$2\pi \to 5\pi/2$ _________________________

A function is called periodic with period p if $f(t+p)=f(t)$. Since $\sin(t+2\pi)=\sin(t)$,

the function $f(t)=\sin(t)$ is ________________ with period ________________

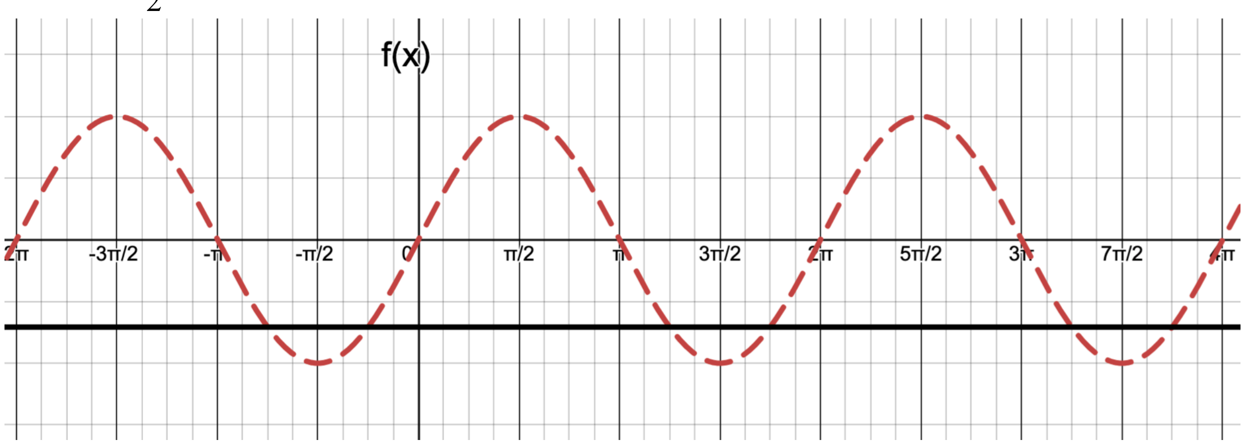
How domain, range, period, even/odd, can be seen on graph.

This type of “sinusoidal wave” can be used to measure many physical phenomena.
Annimation: See https://www.geogebra.org/m/cNEtsbvC



Using the graph to help visualize\ solutions to trig. Equations. As done in an earlier example. solve

$$\sin(t)=-\frac{\sqrt{2}}{2}$$



$f(t) = \cos(t)$

The graph of $f(t) = \cos(t)$ can be generated similarly. In particular, plot the points corresponding to the quadrantal inputs (angles),

t_1	$f(t_1)$
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	

then using the unit circle, consider what ___________ does as t goes from

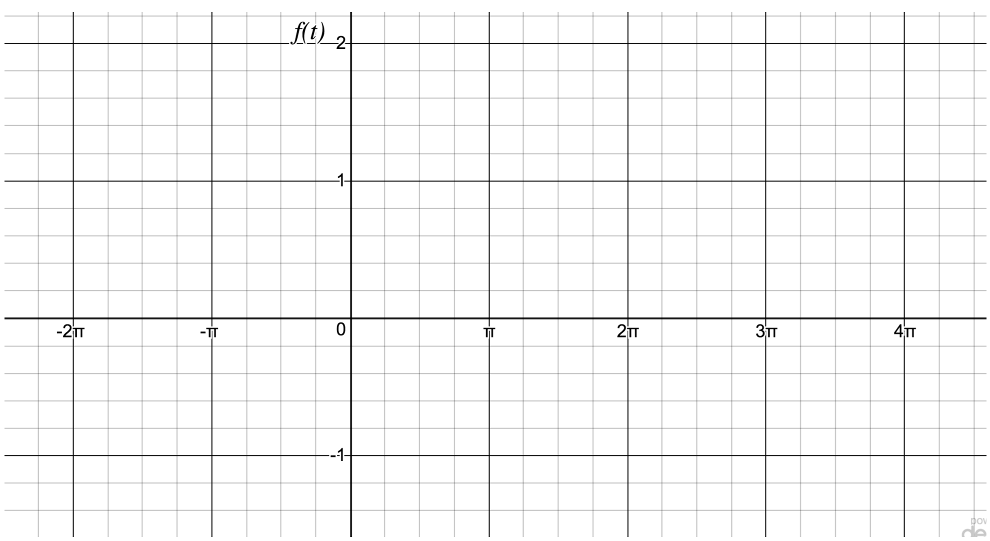
$0 \to \pi/2$

$\pi/2 \to \pi$ _________________________

$\pi \to 3\pi/2$ _________________________

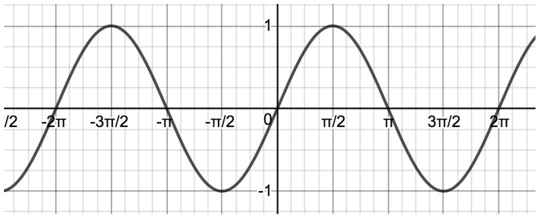
$3\pi/2 \to 2\pi$ _________________________

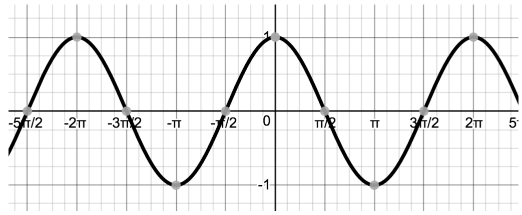
$2\pi \to 5\pi/2$ _________________________



See the animation https://www.geogebra.org/m/cNEtsbvC again

Note: On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points on graph.





______________________ ______________________

Both these graphs are ______________________ with period _________________ and have key points occuring every quadrantal angle or every _____________________

Transformations of the sine and cosine graphs.

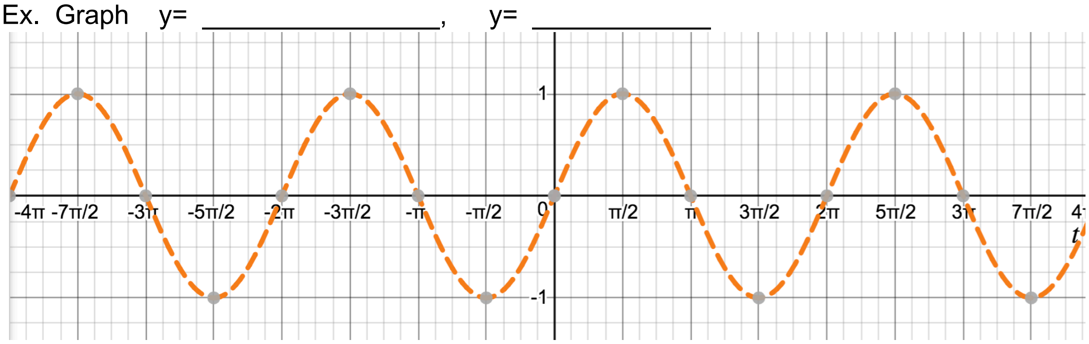
These two graphs can be used as basic graphs together with transformations (review 2.6 as needed).

Vertical Shift

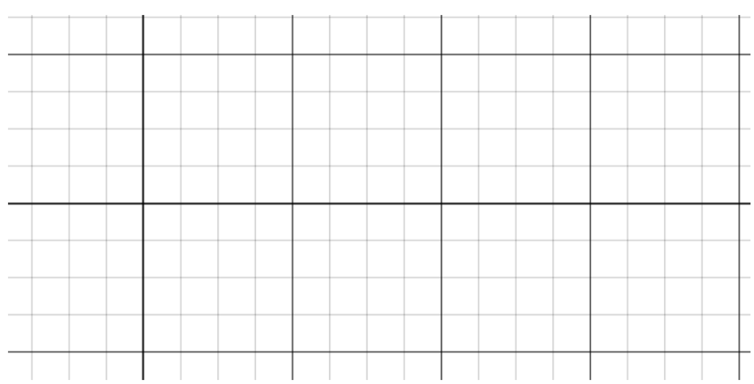
$$f(x)+c$$

$$f(x)-c$$

Ex. Graph y= _______________, y= ____________

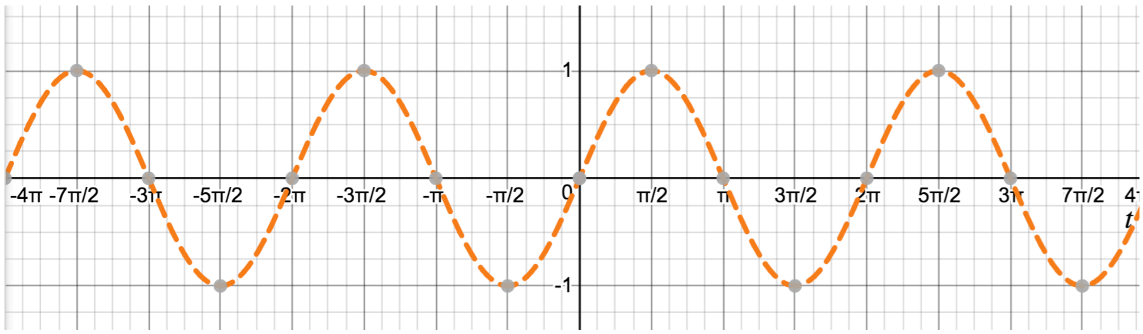


Ideally, eventually, rather than graph the original and then transform it, you would be able picture the transformation in your head to get a starting point, and then use the "quarter period pattern" to generate the rest.

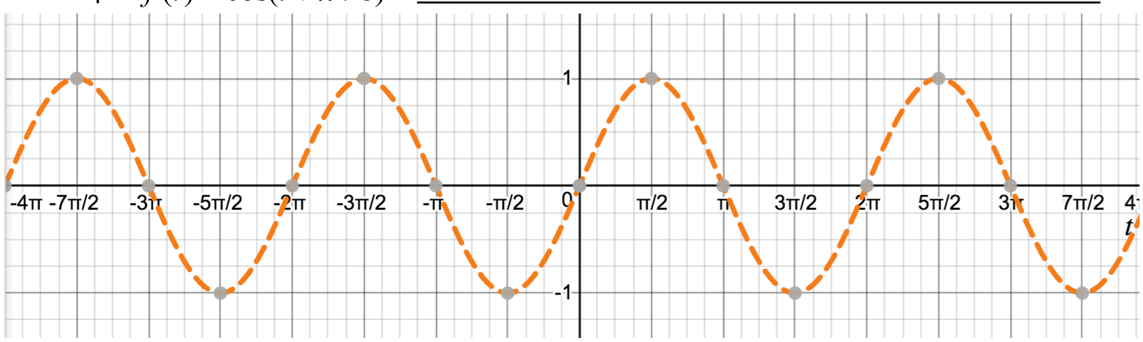


Horizontal Shift	$f(x+c)$ $f(x-c)$	

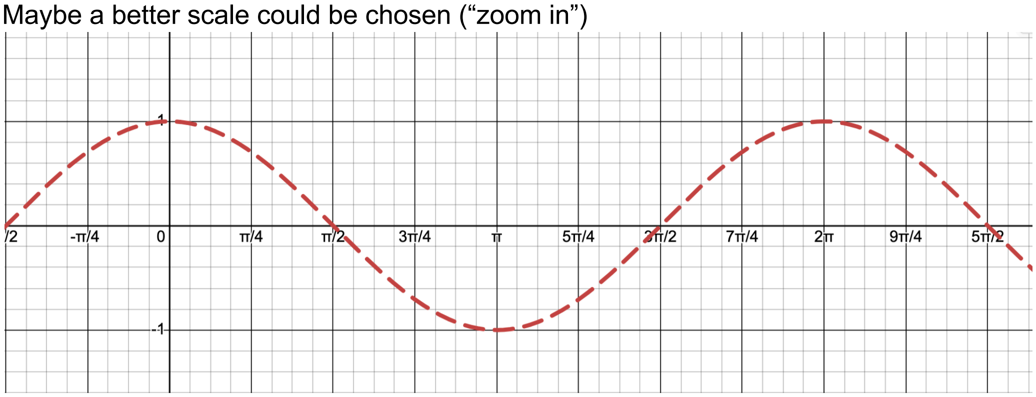
Ex. Graph $f(t)=\cos(t-\pi/4)$ ________________________________



Ex. Graph $f(t)=\cos(t+\pi/8)$ ________________________________

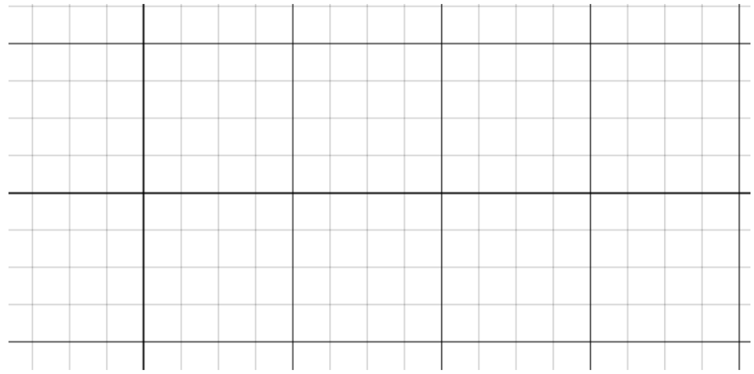


Maybe a better scale could be chosen ("zoom in")



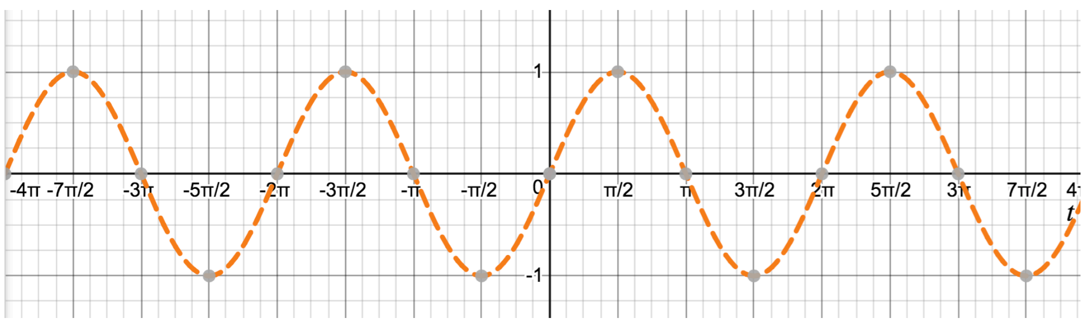
Graph $f(t) = \sin(t - \pi)$

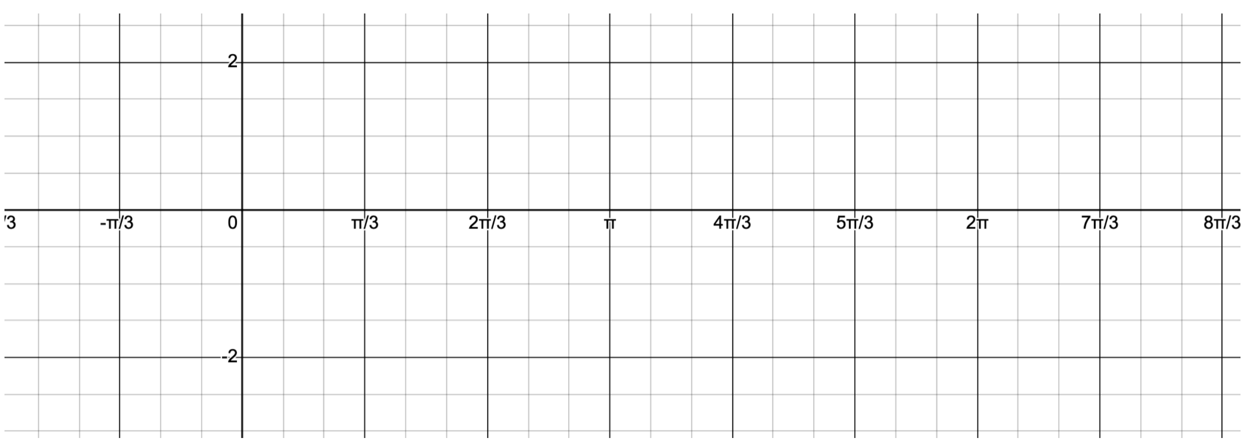
Again, rather than graph the original and then transform it, picture the transformation in your head to get a starting point, and then use the “quarter period pattern” to generate the rest



When graphing a sine or cosine graph, a choice of scale showing multiples of is usually a good choice, but in some cases, a better choice can be made.

Graph $y = \sin\left(t + \frac{\pi}{3}\right)$ ______________________________

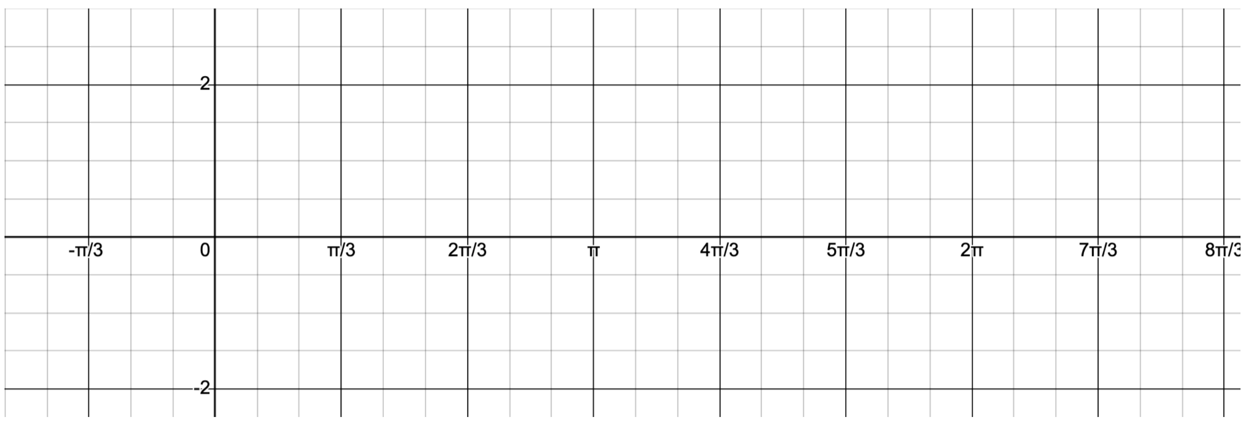




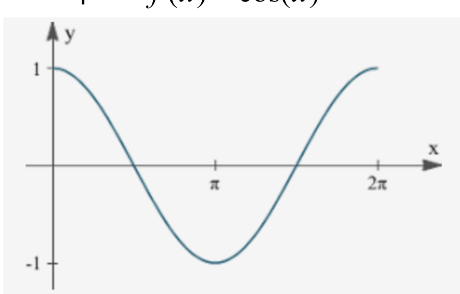
Combining Translations:

How would we graph $y = \sin\left(t + \frac{\pi}{3}\right) + \frac{1}{2}$?______________________________

And ideally, without having to sketch the intermediate stages.

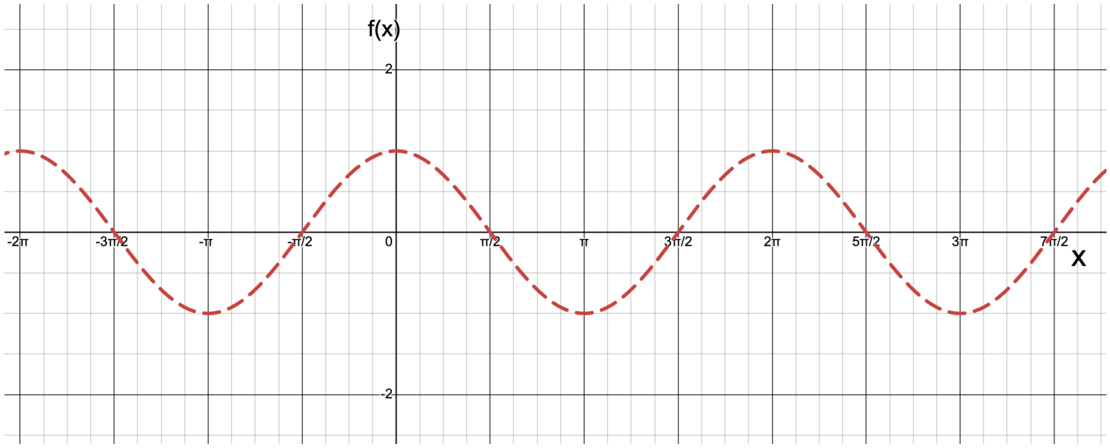


Note: At this point, as a convention, we switch our input from t to x but keep in mind, this x is not the same as the x value of the point on the unit circle.
For example: $f(x) = \cos(x)$

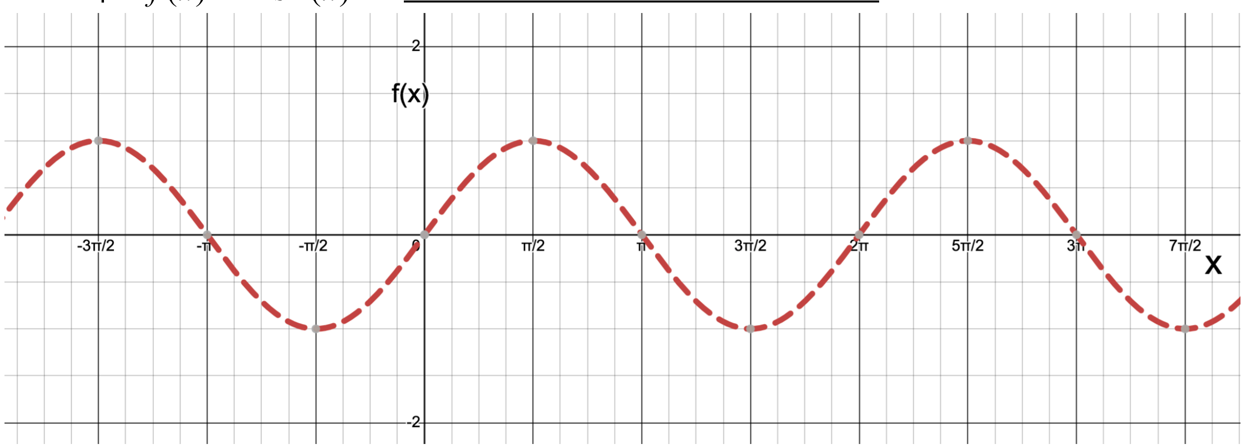


Vertical Stretch/Compress and Reflection. $y = c\,f(x)$

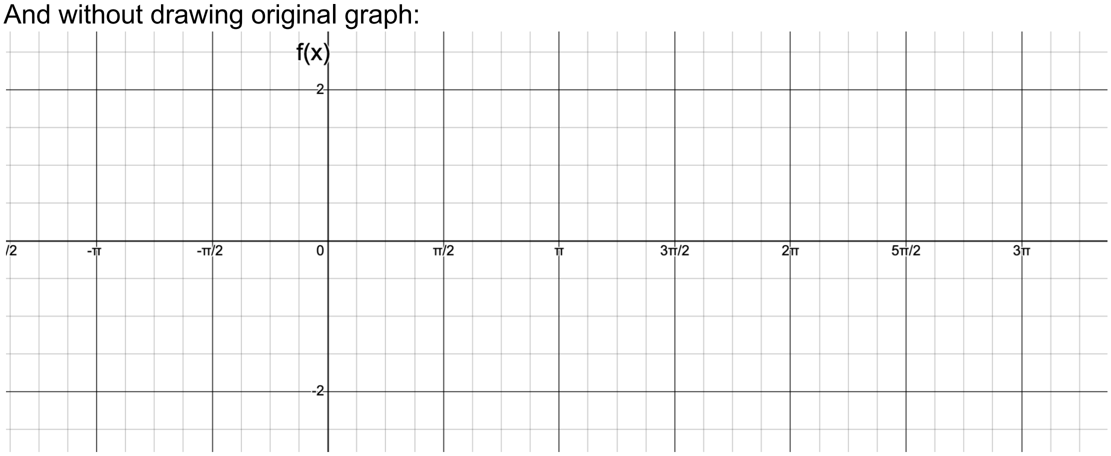
Ex. Graph $f(x) = 2\cos(x)$ $\qquad\qquad$ $f(x) = \frac{1}{2}\cos(x)$



Ex. Graph $f(x) = -2\sin(x)$ ____________________________



And without drawing original graph:

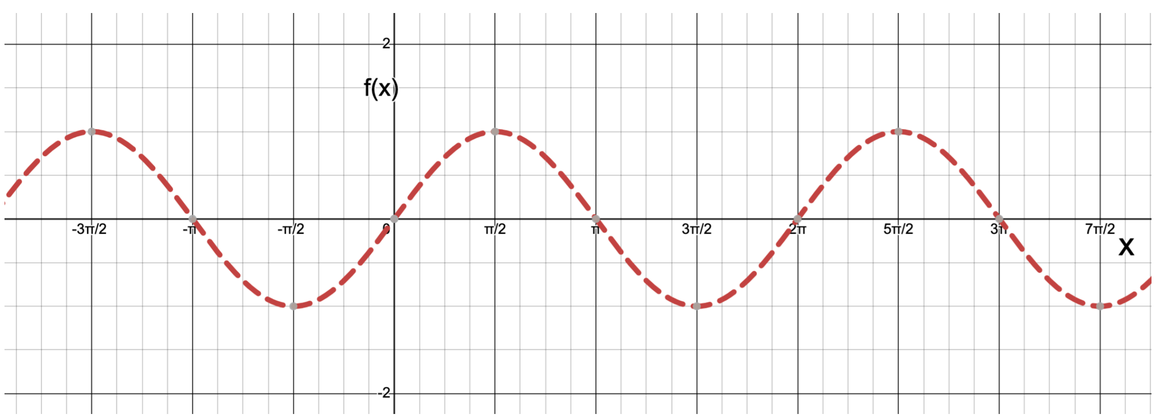


In general, for graphs of the form: $y = a\cos(t)$, $y = a\sin(t)$

5.3ii Graphing the Sine and Cosine Function part (period change)

Horizontal Stretch/Compress and Reflection. $y = f(cx)$

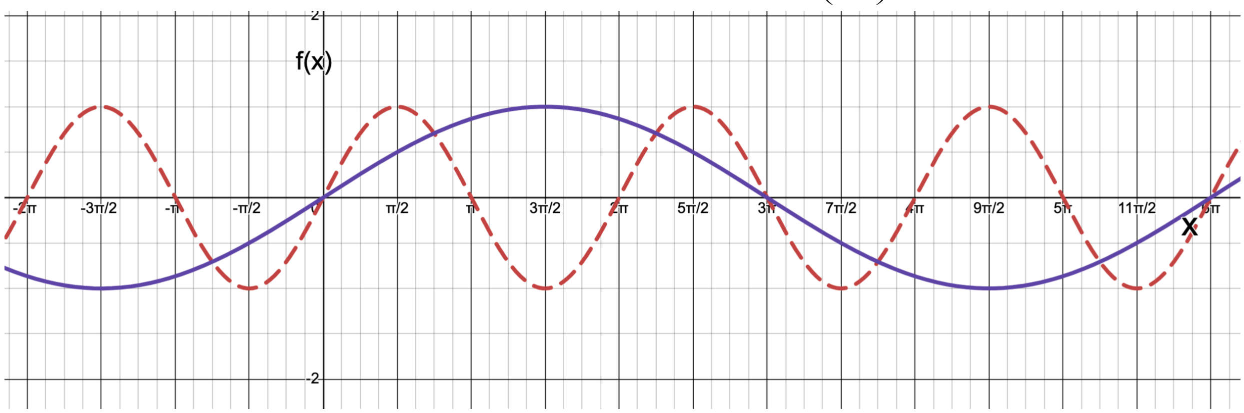
Graph $y = \sin(2x)$ ____



Initially, we might graph this by using our knowledge of horizontal compression or we night simply plot points (note: plotting points is inefficient and should be our last resource.)

Period?___________________________

Thus, the above graph is a horizontal compression whereas $y = \sin\left(\frac{1}{3}x\right)$ is a horizontal stretch.



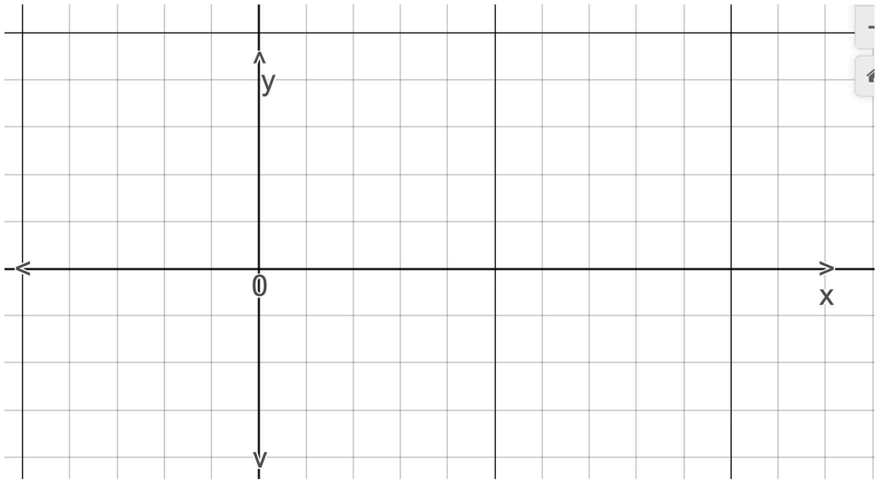
Period ________________?

In general, for graphs of the form: $y = \cos(kx)$, $y = \sin(kx)$ k has the effect of changing the _______________ to ___________________

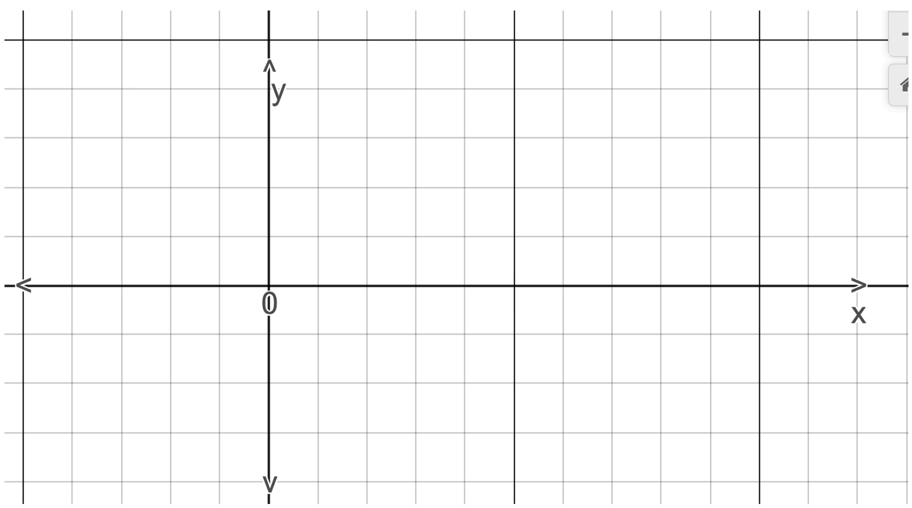
For this type of graph, rather than sketch the original graph and then stretch/compress it, we plan ahead and find the period. Then we break this period into fourths since the key points (lo-zero-hi-zero) occur every one-fourth of the period, and choose our x axis scale accordingly.

Reminder: On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points on graph

Ex: Graph at least one period of________________________ .



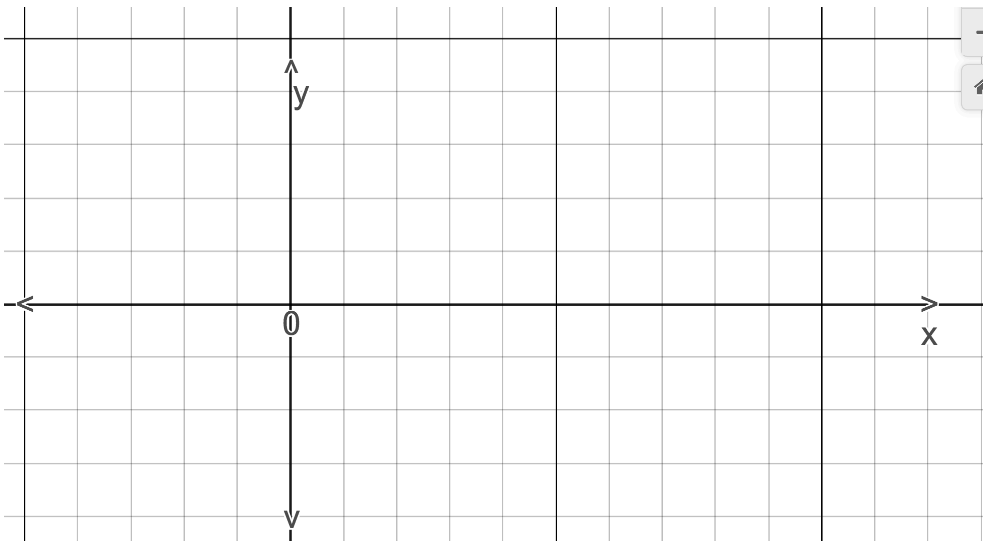
Ex: Graph at least one period of ____________________ .



Combining vertical and horizontal stretch/compress.

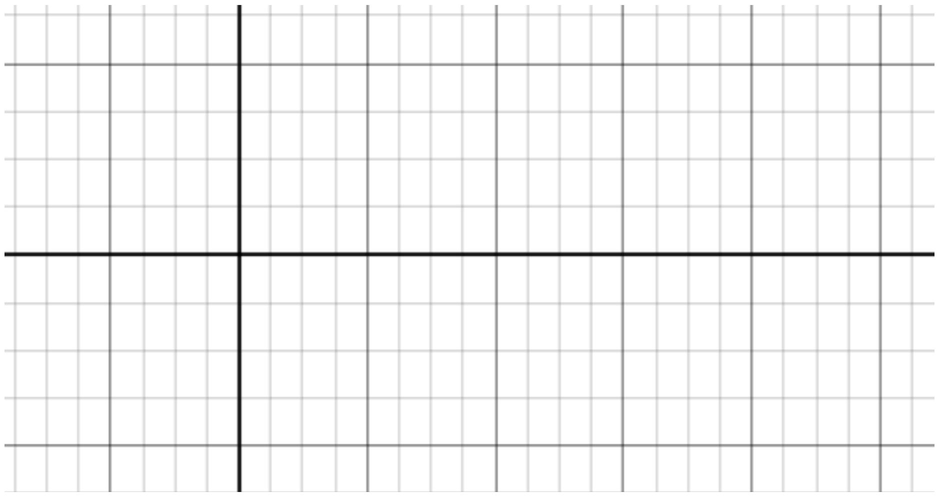
$y = A\cos(kx)$ $y = A\sin(kx)$

Ex:

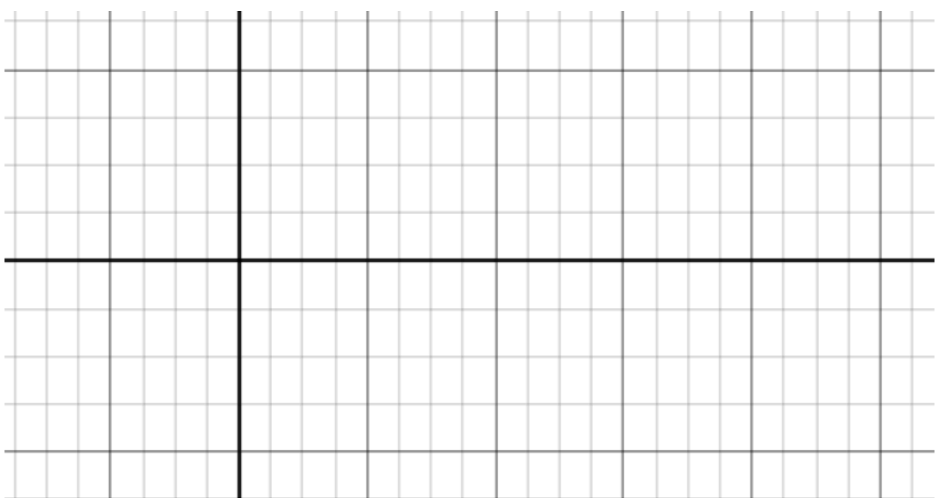


This next example will lead us into the third part of graphing sine and cosine where we put it all together.

Graph $f(x) = 4\sin(2x)$

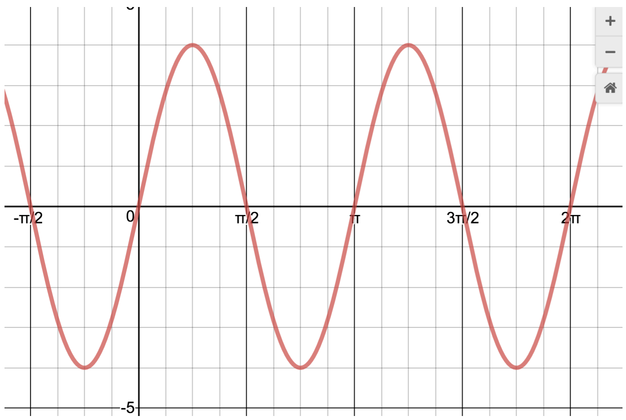


Use the above graph to graph $g(x) = 4\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$



5.3ii Graphing the Sine and Cosine Function part iii : Putting it All Together

Use the graph of $f(x)=4\sin(2x)$ to graph $g(x)=4\sin\left(2\left(x-\frac{\pi}{4}\right)\right)$



Notice, the function $g(x)$ would normally be written ________________

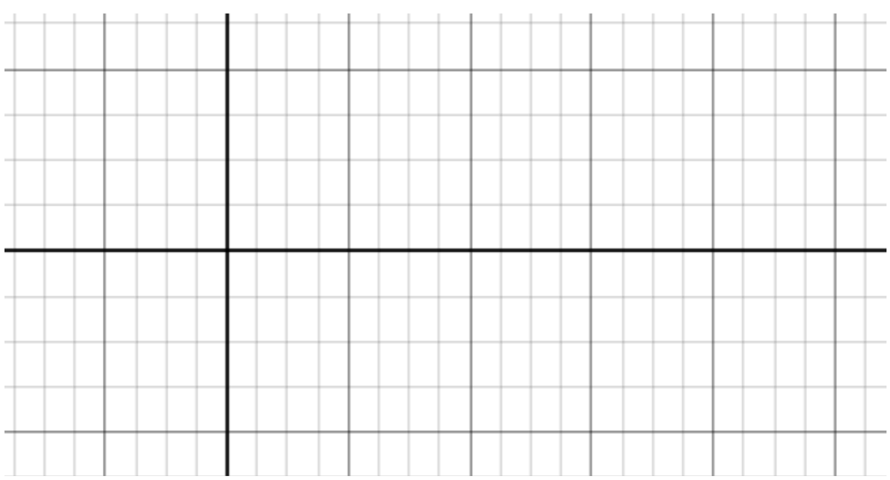
But what was the horizontal shift?_________________________

Note: The horizontal shift is NOT ______________________

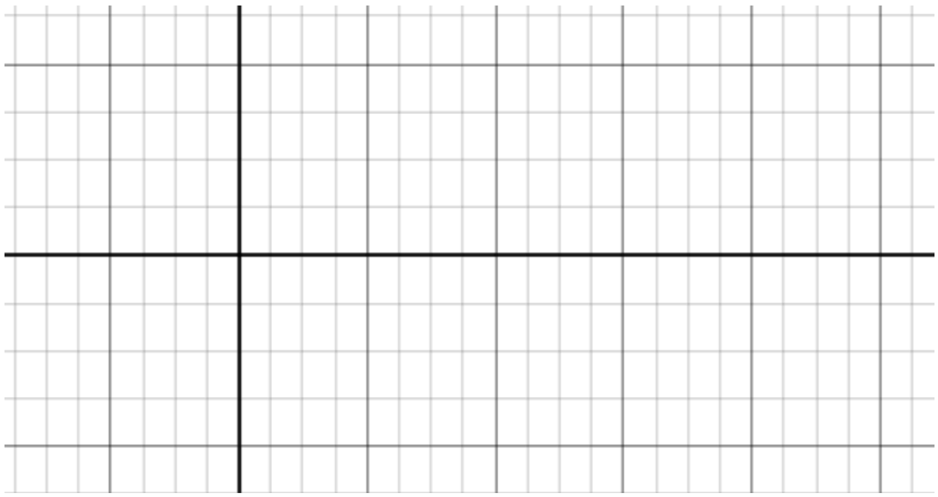
So given $g(x)$___________________ to find the horizontal shift, we have to factor out the 2.

Examples

Graph $f(x)=2\sin\left(3x-\frac{\pi}{4}\right)$



Graph $f(x)=3\cos\left(\pi x+\frac{\pi}{6}\right)$



How would we graph $g(x)=-3\cos\left(\pi x+\frac{\pi}{6}\right)+1$?________________________________

Summarizing $f(x)=a\cos\big(k(x+b)\big)+c$ $\quad$ $f(x)=a\sin\big(k(x+b)\big)+c$